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CUMULATIVE BEHAVIOR OF CONVERGENT SHOCKS WITH DISSIPATION EFFECTS

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1. One-dimensional (spherically or cylindrically symmetric) converging shock waves represent a familiar example of cumulative gasdynamical processes, which play so vital a part in nature and technology [1]. Asymptotic solutions of the converging shock-wave problem in the neighborhood of the center of axis of symmetry are found in the well-known self-similar solutions indicated independently by Guderley [2] and Landau and Stanyukovich [3]. The domain of validity of these solutions depends on the initial and boundary conditions (the simplest of which occur for a cold stationary gas with constant density ρ_0 and a constant-velocity piston), but a self-similar solution is almost always realized in a sufficiently close neighborhood [4]. In this solution, at the instant of "cumulation" of the shock front (usually taken as t = 0) and strictly at the center or axis of symmetry, the velocity of the front ("shock velocity") as well as the pressure and temperature at the front increase without bound: $\dot{r}_f \sim (-t)^{1/k-1} \sim r_f^{1-k} (r_f^k \sim -t)$; $p_f \sim T_f \sim (-t)^{2\left(\frac{1}{k}-1\right)} \sim r_f^{2(1-k)}$, where the self-similarity index $k = k(\gamma) \geqslant 1$ for an isentropic exponent of the gas $\gamma \geqslant 1$ [4, 5]. The self-similar variable, on which depend all the unknown functions of the self-similar solution, has the form $\xi = r/r_f = \xi_0 r^k/(-t)$ in this case, where at the shock front $\xi = 1$ and $r_f = (-t/\xi_0)^{\frac{1}{k}}$. The only arbitrary constant in the self-similar solution has dimensions $cm^{-k}s$ and quantitatively characterizes the "strength" of the initial impulse. The self-similar solution admits continuation in the reflected-shock stage. The cumulative buildup of energy between the shock front $\xi = 1$ and an arbitrary value of the variable $\xi^* > 1$ behind the front (ξ^* normally coincides with a singular ξ-line that nowhere intersects the C-characteristics directed toward the shock front) is characterized by the following dependence on the radius of the shock front: $E_a \sim r_f^{5-2k}$ (spherical case) or $E_a \sim r_f^{4-2k}$ (cylindrical case) [4]. As a result of the cumulative process, the energy of this region decreases as $r_f \to 0$ more slowly than r_f^3 (spherical case) or r_f^2 (cylindrical case), because the self-similarity index $k > 1$ (or $\gamma > 1$).

In this article we investigate the constraints imposed on the "cumulation" parameters of the self-similar solution (with inclusion of the reflected-shock stage) due to dissipation effects. These effects clearly become significant when the effective mean free path of the investigated gas particles is commensurate with the radius of the shock front, $l_s \sim r_f$. The allowance for dissipation effects, first of all, shows that all the hydrodynamic and thermodynamic variables are bounded and, second, yields very general expressions for the maximum cumulation parameters, which are determined simultaneously with the characteristics of the self-similar solution and dissipation effects. Of course, the cumulation parameters can actually also be affected by the deviations of the motion of the gas from one-dimensionality in connection with the singularities due to the well-known instability of converging shock waves [6, 7]. We realize that these deviations are rendered inconsequential by the sufficiently symmetric initial and boundary conditions of the problem. Accordingly, violations of the one-dimensional symmetry of the motion can not only attenuate the cumulation parameters, but can amplify them as well, as shown by the examples of two-dimensional cumulative motions in the case of a plasma focus and a current sheet [8, 9].

For sufficiently strong converging shock waves, in general, the most interesting case is a fully ionized plasma. If a high-temperature plasma generated in a cumulation zone has a sufficiently high density, its motion is described by the system of two-temperature gasdynamic equations with well-known dissipation effects, which in this case are associated with

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the ionic viscosity and thermal conductivity, the electronic thermal conductivity, and finally the energy exchange between ions and electrons [10-12]. In the classical problem of the structure of a plane stationary shock front it has been shown [4, 13] that the inclusion of these dissipation effects imparts a continuous structure to the front for any shock strength. The existence of a continuous solution in this case implies boundedness of the derivatives of all variables.* The indicated system of equations has been used to carry out numerical calculations of complex plasma flows in cylindrically or axially symmetric geometries (comprising a two-dimensional problem in the latter case), which are inherent in cylindrical or non-cylindrical Z-pinch [14, 12, 15]. In particular, the cumulation parameters of converging shock waves in the pinch axial zone are obtained in these calculations. Arrival at a self-similar solution is observed in this situation, but the particular boundary conditions and other parameters of the system are not favorable to the general occurrence of such a solution. A self-similar solution for a converging cylindrically symmetric shock wave in the axial zone of a plasma has been specially studied [16]. The present article is essentially a continuation and generalization of [16]. Here we derive general relations for the maximum cumulation parameters, but not exclusively in the case of a plasma or in the cylindrically symmetric problem. We include in the study a different set of dissipation coefficients, corresponding formally, in accordance with [17], to a gas composed of particles with arbitrary (but of the power-law type) interaction forced between particles, and we take into consideration a form of spherical geometry of the system.

It is quite clear from the foregoing considerations how to construct such a solution for any kind of dissipation terms in the gasdynamic system of equations for any equation of state of the gas. The nontrivial problem of the validity of the gasdynamic description of the gas near the axis or center of symmetry is completely solvable only by comparison with the corresponding kinetic treatment. However, the validity of the gasdynamic approximation with regard for the above-mentioned dissipation effects to the problem of the structure of the front of a shock wave or, more precisely, a collision shock wave [13] lends credibility to the present investigation. It can even be stated that the disregarded kinetic effects only slightly alter the quantitative estimates of the cumulation parameters without affecting the order of magnitude of the resulting variables and general relations.

2. In the neighborhood of the center or axis of symmetry of the system we write the one-dimensional gasdynamic system of equations with regard for the effects of viscosity and thermal conduction of the gas where the corresponding coefficients have a nonlinear behavior ($\nu = 1$ for the cylindrical case; $\nu = 2$ for the spherical case) [14, 18]:

$$\frac{d\rho}{dt} + \rho\frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu v) = 0, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial r}; \tag{2.1}$$

$$\rho\frac{dv}{dt} + \frac{\partial}{\partial r}(a\rho T) = \frac{4}{3}\frac{\partial}{\partial r}\left(\eta\frac{\partial v}{\partial r}\right) + \frac{2}{3}\nu\left[2\eta\frac{\partial}{\partial r}\left(\frac{v}{r}\right) - \frac{v}{r}\frac{\partial\eta}{\partial r}\right]; \tag{2.2}$$

$$\frac{a}{\gamma-1}\rho\frac{dT}{dt} - aT\frac{d\rho}{dt} = \frac{4}{3}\eta\left[\left(\frac{\partial v}{\partial r}\right)^2 - \nu\frac{v}{r}\left(\frac{\partial v}{\partial r} - \frac{3-\nu}{2}\frac{v}{r}\right)\right] + \frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\varkappa\frac{\partial T}{\partial r}\right), \tag{2.3}$$

where a is the gas constant ($p = a\rho T$ and $a = k/m$ in the case of a simple gas comprising identical particles of mass m), η is the viscosity coefficient, and $\varkappa$ is the thermal conductivity. Let these nonlinear coefficients depend only on the temperature T in accordance with the same power-law relation:

$$\eta = \eta_0(aT)^s, \quad \varkappa = \varkappa_0(aT)^s. \tag{2.4}$$

These relations are obtained for a simple gas of identical particles interacting with a mutual force $F \sim r^{-m}$, so that $s = 1/2 + 2/(m-1)$ [17]. In this case the Prandtl number $\Pr \sim \eta/\varkappa = \text{const}$, this constant depending only slightly on the power m in the force law. In the special case of a plasma, where $F \sim r^{-2}$, i.e., m = 2 and s = 5/2, the well-known plasma relation [19] follows at once from (2.4). In a plasma, however, it is necessary to treat a mixture of ions and electrons, rather than a simple gas, whereupon the initial system of equations (2.1)-(2.3) is somewhat more complicated. This elaboration of the initial system is carried out below in dimensionless form. We take the isentropic exponent $\gamma = 5/3$ in (2.3), limiting the discussion to a monatomic gas.

The system of equations (2.1)-(2.3) must be reduced to dimensionless form by introducing dimensionless variables according to the relations

$$\begin{gathered}\sigma = \rho/\rho_0, \quad x = r/r_0, \quad \tau = t/t_0, \quad u = v/v_0 \quad (v_0 = r_0/t_0),\\ \Pi = \frac{p}{p_0}\left(p_0 = \rho_0\frac{r_0^2}{t_0^2}\right), \quad \Theta = \frac{T}{T_0}\left(T_0 = \frac{1}{a}\frac{r_0^2}{t_0^2}\right), \quad \Pi = \sigma\Theta.\end{gathered} \tag{2.5}$$

The system of units (or scales) in relations (2.5) is based on the three dimensioned quantities r_0, t_0, and ρ_0. Substituting these quantities into Eqs. (2.1)-(2.3), we obtain a certain variation of the conventional dimensionless system of equations

*It is expected that in this case, in any neighborhood of the center or axis of symmetry these derivatives will remain bounded and the functions themselves all therefore remain finite.

$$\frac{d\sigma}{d\tau}+\sigma\frac{1}{x^{\nu}}\frac{\partial}{\partial x}(x^{\nu}u)=0,\quad \frac{d}{d\tau}=\frac{\partial}{\partial\tau}+u\frac{\partial}{\partial x};\tag{2.6}$$

$$\sigma\frac{du}{d\tau}+\frac{\partial}{\partial x}(\sigma\Theta)=\frac{1}{\mathrm{Re}}\left\{\frac{\partial}{\partial x}\left(\Theta^{s}\frac{\partial u}{\partial x}\right)+\nu\left[\Theta^{s}\frac{\partial}{\partial x}\left(\frac{u}{x}\right)-\frac{1}{2}\frac{u}{x}\frac{\partial\Theta^{s}}{\partial x}\right]\right\};\tag{2.7}$$

$$\frac{3}{2}\sigma\frac{d\Theta}{d\tau}-\Theta\frac{d\sigma}{d\tau}=\frac{\Theta^{s}}{\mathrm{Re}}\left[\left(\frac{\partial u}{\partial x}\right)^{2}-\nu\frac{u}{x}\left(\frac{\partial u}{\partial x}-\frac{3-\nu}{2}\frac{u}{x}\right)\right]+\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{1}{x^{\nu}}\frac{\partial}{\partial x}\left(x^{\nu}\Theta^{s}\frac{\partial\Theta}{\partial x}\right),\tag{2.8}$$

where the dimensionless numbers Re and Pr, according to (2.4) and (2.5), are given by the expressions

$$\frac{1}{\mathrm{Re}}=\frac{4}{3}\frac{\eta_0}{\rho_0}\frac{r_0^{2s-2}}{t_0^{2s-1}},\quad \frac{1}{\mathrm{Pr}}=\frac{3}{4}\frac{\varkappa_0}{a\eta_0}.\tag{2.9}$$

Up to now the length and time scales (r_0 and t_0) have been arbitrary (the density scale must naturally coincide with the constant density of the undisturbed gas). We now relate them to the parameters of the self-similar solution; we assume that their interrelationship is such as to satisfy the equation for the shock front $\xi_0 r_f^k t_f^{-1}=1$, i.e.,

$$\xi_0 r_0^k t_0^{-1}=1.\tag{2.10}$$

For the final determination of r_0 and t_0 we require that the dimensionless number Re from relations (2.9) be equal to unity, i.e.,

$$\frac{4}{3}\frac{\eta_0}{\rho_0}\frac{r_0^{2s-2}}{\xi_0^{2s-1}r_0^{k(2s-1)}}=\frac{4}{3}\frac{\eta_0}{\rho_0}\frac{r_0^{2s(1-k)+k-2}}{\xi_0^{2s-1}}=1,\tag{2.11}$$

whence we obtain the units r_0 and t_0 in explicit form [with regard for (2.10)]:

$$r_0=\left(\frac{3}{4}\frac{\rho_0}{\eta_0}\xi_0^{2s-1}\right)^{\frac{1}{2s(1-k)+k-2}},\quad t_0=\left(\frac{3}{4}\frac{\rho_0}{\eta_0}\xi_0^{2s-2}\right)^{\frac{1}{2s(1-k)+k-2}}.\tag{2.12}$$

Thus, with r_0 and t_0 chosen according to (2.12), we must put Re = 1 in Eqs. (2.7) and (2.8), retaining the number Pr independent of the scales r_0 and t_0, as determined from (2.9). The dimensionless number Pr, as mentioned above, depends only slightly on the power m in the force law, and if we neglect this dependence (obtained in the higher approximations of the Chapman–Enskog method), then in the first approximation [17],

$$\frac{1}{\mathrm{Pr}}=\frac{3}{4}\frac{\varkappa_0}{a\eta_0}=\frac{45}{16}\ ^{*}.\tag{2.13}$$

It may be assumed that the system of equations (2.6)-(2.8) with Re = 1 no longer contains any parameters. All that remains is to formulate the boundary conditions in order to complete the statement of the problem of the motion of the gas near the axis or center of symmetry for a converging self-similar shock wave.

3. It can be shown that the dimensionless number Re $\sim r_0/l_s$, where l_s is the effective mean free path of the particles. Thus, in order of magnitude the viscosity coefficient $\eta\sim\rho l_s v_T$, where $v_T\sim(aT)^{1/2}$ is the average thermal velocity of the particles. We form the ratio r_0/l_s and once again make use of the definitions of the dimensionless quantities (2.5) and relation (2.4):

$$\frac{r_0}{l_s}\sim\frac{r_0\rho v_T}{\eta}\sim\frac{r_0\rho\,(aT)^{\frac{1}{2}}}{\eta_0\,(aT)^{s}}\sim\frac{r_0\rho_0}{\eta_0\,(aT_0)^{s-\frac{1}{2}}}\sim\frac{\rho_0}{\eta_0}\frac{t_0^{2s-1}}{r_0^{2s-2}},\tag{3.1}$$

i.e., according to (2.9), $r_0/l_s\sim$ Re. Consequently, by taking Re = 1 we make the scale r_0 in (2.12) equal in order of magnitude to the effective mean free path of the particles. This means that the outer boundary of the domain of solution of the dimensionless system of equations (2.6)-(2.8) (with Re = 1) must be taken as $x=X_0(\tau)\gtrsim 1$. For such a boundary it is logical to specify a Lagrangian trajectory and the pressure on it, borrowed from the self-similar solution. We thus specify the boundary conditions

*In the case of rigid molecules ($m\to\infty$) the factor 1.0088 rises in the second approximation, i.e., $\mathrm{Pr}^{-1}=2.84$, whereas for the case of Coulomb forces (m = 2) the corresponding factor in the second approximation is 1.0854, i.e., $\mathrm{Pr}^{-1}=3.05$ [17].

$$u = 0, \ \partial\Theta/\partial x = 0, \ x = 0; \tag{3.2}$$

$$dX_0/d\tau = u, \quad \partial\Theta/\partial x = 0, \quad \sigma\Theta - \left(\frac{\partial u}{\partial x} - \frac{1}{2}\nu\frac{u}{x}\right) = \Pi(\tau), \quad x = X_0(\tau).$$

In (3.2) the pressure from the self-similar solution $\Pi(\tau)$, strictly speaking, is taken to be equal to the radial component of the momentum flux, rather than to the pressure per se. However, the additional viscosity term in the parentheses ($\sim\sigma'_{rr}$) is very small if the Lagrangian trajectory is chosen in the way described above, namely sufficiently far from the center or axis of symmetry.

It can be shown that $\Pi(\tau)$ is a universal function that does not depend on the dimensioned parameters of the self-similar solution ξ_0 and ρ_0 in the adopted system of units (2.5). Thus, according to [4, 5], the self-similar solution has the form $v = (r/t)U(\xi)$, $p = \rho_0(r^2/t^2)P(\xi)$, where $\xi = -\xi_0 r^k t^{-1}$, and in the dimensionless variables (2.5) with the use of relation (2.10) between the scales r_0 and t_0

$$u = \frac{1}{r_0}\frac{r_0}{t_0}\frac{x}{\tau}U(\xi) = \frac{x}{\tau}U(\xi), \quad \Pi = \frac{1}{P_0}\rho_0\frac{r_0^2}{t_0^2}\frac{x^2}{\tau^2}P(\xi) = \frac{x^2}{\tau^2}P(\xi), \tag{3.3}$$
$$\xi = -\xi_0 r_0^k t_0^{-1} x^k \tau^{-1} = -x^k\tau^{-1},$$

where the specificity of the choice of (2.10) is manifested only in the invariance of the self-similar variable ξ. The dimensionless functions $U(\xi)$, $P(\xi)$, etc., themselves turn out to be invariant in the new system of units. We write the elementary equation for an arbitrary Lagrangian trajectory $X_1(\tau)$:

$$\frac{dX_1}{d\tau} = \frac{x_f}{\tau}\xi^{\frac{1}{k}}U(\xi), \quad \xi = -X_1^k\tau^{-1}, \quad \tau > \tau_0 = -(X_1^0)^k, \tag{3.4}$$

where the initial value $X_1^0 \gg 1$ corresponds to the instant of passage of the shock front, i.e., at $\tau = \tau_0$, $X_1^0 = x_f(\tau_0)$ $(1 = -x_f^k\tau_f^{-1}$ is the equation for the shock front in dimensionless coordinates). On the trajectory $X_1(\tau)$ the function $\Pi(\tau)$ entering into the boundary conditions (3.2), according to (3.3), has the form

$$\Pi(\tau) = \frac{X_1^2(\tau)}{\tau^2}P\left(-\frac{X_1^k(\tau)}{\tau}\right), \tag{3.5}$$

where $X_1(\tau)$ is given by Eq. (3.4). It is evident from (3.4) and (3.5) that the function $\Pi(\tau)$ is of a universal nature. We note that in (3.2) the trajectory $X_0(\tau)$ differs in principle from the initial self-similar trajectory $X_1(\tau)$ insofar as the function $X_0(\tau)$ deduced from the solution of the system of equations (2.6)-(2.8) with Re = 1 is now influenced by dissipation effects. However, for a sufficiently large $X_1^0 \gg 1$ the dissipation effects become insignificant, and $X_0(\tau) \simeq X_1(\tau)$. The closeness of these functions can be regarded as a quantitative criterion of the correct choice of the outer Lagrangian trajectory. It is quite clear that if $(X_1^0)'$ is sufficiently large in the above-stated sense, then the solution of Eqs. (2.6)-(2.8) with any other value $(X_1^0)'' > (X_1^0)'$ will not differ in any way from their solution with the previous value $(X_1^0)'$. This fact implies the existence of a unique asymptotic solution of the problem in the neighborhood of the center or axis of symmetry.*

4. Let us assume that the indicated problem has been solved by the numerical method. It is remarkable that for a fixed particle-interaction law [power s and number Pr in Eqs. (2.7) and (2.8)] in the given geometry [power in Eqs. (2.6)-(2.8)] for the given self-similar solution [self-similarity index in relations (3.4), (3.5) and the boundary condition (3.2)] it does not depend on any other parameters. The physical considerations set forth in No. 1 suggest that this solution for the quantities σ, u, and Θ will be bounded for a finite value of the function $\Pi(\tau)$ in the boundary condition. Let us suppose that the maximum values u_{max}, Θ_{max}, Π_{max} have been determined. We express the corresponding characteristic values of the velocity u'_{max}, temperature Θ'_{max}, and pressure Π'_{max} for the complete gasdynamical problem of a converging shock wave in the self-similar case in terms of the foregoing maximum values.

For the scales of the quantities in the complete problem it is natural to include the characteristic pressure p'_0 along with the initial density $\rho'_0 = \rho_0$ and the radius of the system r'_0. This additional characteristic literally determines the

*We call attention to the condition $\partial\Theta/\partial x = 0$ (at x = 0), which actually predetermines the finiteness of the temperature at the point x = 0, but not of any other quantities (density and velocity) (cf. an isothermal discontinuity). However, a direct numerical solution [16], albeit carried out in one special case, shows that all the hydrodynamic and thermodynamic variables in the given statement of the problem turn out to be finite. This result is consistent with our assumptions in connection with the existence of a continuous solution of the problem of the structure of the shock front (see the preceding footnote).

"strength" of the initial impulse. Then the corresponding dimensionless variables [by analogy with (2.5)] have the form

$$\sigma' = \frac{\rho}{\rho_0'} \quad (\rho_0' = \rho_0), \quad x' = \frac{r}{r_0'}, \quad \Pi' = \frac{p}{p_0'}, \quad \tau' = \frac{t}{t_0'}\left(t_0' = \frac{r_0'}{(p_0'/\rho_0)^{\frac{1}{2}}}\right),$$
$$u' = \frac{v}{v_0'} \quad \left(v_0' = \frac{r_0'}{t_0'} = \left(\frac{p_0'}{\rho_0}\right)^{\frac{1}{2}}\right), \quad \Theta' = \frac{T}{T_0'} \quad \left(T_0' = \frac{1}{a}\frac{p_0'}{\rho_0}\right). \tag{4.1}$$

The constant ξ_0 in the self-similar solution is expressed in these variables by the equation

$$\xi_0 = -r_f^{-k} t_f = -r_0'^{-k} t_0' x_f'^{-k} \tau_f' = \xi_0' (r_0')^{1-k} \left(\frac{p_0'}{\rho_0}\right)^{-\frac{1}{2}}, \quad \xi_0' = -x_f'^{-k} \tau_f', \tag{4.2}$$

in which ξ_0' is the constant of the self-similar solution in the units (4.1).

The characteristic maximum temperature Θ_{max}' can then be expressed in terms of the calculated quantity Θ_{max} as follows with regard for (2.5), (4.1), (2.10), and (2.12):

$$\Theta_{max}' = \Theta_{max} \frac{T_0}{T_0'} = \Theta_{max} \frac{r_0^2}{t_0^2} \frac{\rho_0}{p_0'} = \Theta_{max} \frac{r_0^{2(1-k)}}{\xi_0^2} \frac{\rho_0}{p_0'} =$$
$$= \Theta_{max} \frac{1}{p_0'} \frac{\rho_0}{\xi_0^2} \left(\frac{3}{4} \frac{\rho_0}{\eta_0} \xi_0^{2s-1}\right)^{\frac{2(1-k)}{2s(1-k)+k-2}} = \Theta_{max} \frac{1}{p_0'} \left(\frac{3}{4\eta_0}\right)^{\frac{2(1-k)}{2s(1-k)+k-2}} \xi_0^{\frac{2}{2s(1-k)+k-2}} \rho_0^{\frac{k+2s(k-1)}{2s(1-k)+k-2}}. \tag{4.3}$$

Now by analogy with (3.1) we calculate the ratio of a certain effective mean free path $l_s^* = \frac{4}{3}\eta_0 (aT_0')^{s-\frac{1}{2}} \rho_0^{-1} = \frac{4}{3}\eta_0 \left(\frac{p_0'}{\rho_0}\right)^{s-\frac{1}{2}} \rho_0^{-1}$ to the length scale r_0', denoting it by the symbol α:

$$\alpha = \frac{l_s^*}{r_0'} = \frac{4}{3} \frac{\eta_0}{\rho_0 r_0'} \left(\frac{p_0'}{\rho_0}\right)^{s-\frac{1}{2}} = \frac{4}{3}\eta_0 \frac{p_0'^{\,s-\frac{1}{2}}}{\rho_0^{s+\frac{1}{2}} r_0'}. \tag{4.4}$$

The parameter α from (4.4) determines the role of dissipation effects and therefore occurs as a coefficient in all the dissipation terms of the system of equations (2.1)-(2.3) written in the units (4.1). For the more complex plasma case this fact has been proved in [12]. It is clear that only under the condition $\alpha \ll 1$ is it possible to have a self-similar solution for a converging shock wave, where the characteristics of this solution, the constant ξ_0' from (4.2) in particular, do not depend on the dimensionless parameter α. Then, substituting the constant ξ_0 from (4.2) into (4.3) and using the definition (4.4) for the parameter α, we finally obtain

$$\Theta_{max}' = \Theta_{max} (\xi_0')^{\frac{2}{2s(1-k)+k-2}} \alpha^{\frac{2(k-1)}{2s(1-k)+k-2}}. \tag{4.5}$$

We supplement (4.5) with the corresponding expressions for Π_{max}' and u_{max}':

$$\Pi_{max}' = \Pi_{max} (\Theta_{max}'/\Theta_{max}), \quad u_{max}' = u_{max} (\Theta_{max}'/\Theta_{max})^{1/2}. \tag{4.6}$$

Of course, expressions (4.5) and (4.6) interrelate not only the maximum values of Π', Θ', and u' with the quantities Π, Θ, and u, but also any values of these quantities for an arbitrary time t and point in space r. It is important to note that the quantities Θ_{max}', Π_{max}', and u_{max}' themselves at once give the degree of cumulation in the converging shock wave, because in the system of units (4.1) all the characteristic quantities are of the order of unity. The constant $\xi_0' \sim 1$.

To illustrate the general relation (4.5) we consider the special cases of Coulomb (m = 2, s = 5/2) and rigid ($m \to \infty$, s = 1/2) particles for a spherically symmetric converging shock wave (ν = 2), such that, according to [3, 5], for γ = 5/3 the index k = 1.453. Accordingly, from (4.5) we obtain two dependences on the parameter α, which is indicated in parentheses for the indicated cases:

$$\Theta_{max}' \sim \Theta_{max}\alpha^{-0.322}\left(\alpha \sim \frac{p_0'^2}{\rho_0^3 r_0'}\right); \quad \Theta_{max}' \sim \Theta_{max}\alpha^{-0.906}\left(\alpha \sim \frac{1}{\rho_0 r_0'}\right).$$

5. In the case of a fully ionized ideal plasma with singly charged ions (Z = 1) the initial equation with all essential dissipation effects included have the following form instead of (2.6)-(2.8) [14, 16]:

$$\sigma\frac{du}{d\tau}+\frac{\partial}{\partial x}[\sigma(\Theta_i+\Theta_e)]=\frac{\partial}{\partial x}\left(\Theta_i^{5/2}\frac{\partial u}{\partial x}\right)+\nu\left[\Theta_i^{5/2}\frac{\partial}{\partial x}\left(\frac{u}{x}\right)-\frac{1}{2}\frac{u}{x}\frac{\partial\Theta_i^{5/2}}{\partial x}\right]; \tag{5.1}$$

$$\frac{3}{2}\sigma\frac{d\Theta_i}{d\tau}-\Theta_i\frac{d\sigma}{d\tau}=\Theta_i^{5/2}\left[\left(\frac{\partial u}{\partial x}\right)^2-\nu\frac{u}{x}\left(\frac{\partial u}{\partial x}-\frac{3-\nu}{2}\frac{u}{x}\right)\right]+ \\ +\frac{3.1}{x^\nu}\frac{\partial}{\partial x}\left(x^\nu\Theta_i^{5/2}\frac{\partial\Theta_i}{\partial x}\right)-5.4\left(\frac{m}{M}\right)^{1/2}\sigma^2\frac{\Theta_i-\Theta_e}{\Theta_e^{3/2}}; \tag{5.2}$$

$$\frac{3}{2}\sigma\frac{d\Theta_e}{d\tau}-\Theta_e\frac{d\sigma}{d\tau}=1.76\left(\frac{M}{m}\right)^{1/2}\frac{1}{x^\nu}\frac{\partial}{\partial x}\left(x^\nu\Theta_e^{5/2}\frac{\partial\Theta_e}{\partial x}\right)+5.4\left(\frac{m}{M}\right)^{1/2}\sigma^2\frac{\Theta_i-\Theta_e}{\Theta_e^{3/2}}. \tag{5.3}$$

The equations (5.2) and (5.3) for the ion and electron temperatures Θ_i and Θ_e include the ratio of the ion mass M to the electron mass m. The ion-electron energy exchange term is proportional to $(m/M)^{1/2}$, and the electronic thermal conduction term is inversely proportional to $(m/M)^{1/2}$. The system of equations for a two-temperature fully ionized plasma (2.6), (5.1)-(5.3) is written in the dimensionless units (2.5); for both temperatures the scale is the same, T_0, $\Pi=\sigma(\Theta_i+\Theta_e)$, and a = k/M. The length and time scales r_0 and t_0 in this case are given by relations (2.12). In place of the number Pr for a simple gas from (2.13), we now have several dimensionless numbers characterizing the ionic thermal conduction (Pr_i^{-1} = 3.1), the electronic thermal conduction $\left(\mathrm{Pr}_e^{-1}=1.76\,(M/m)^{\frac{1}{2}}\right)$, and finally the ion-electron energy exchange $\left(C_{ei}^{-1}=5.4\,(m/M)^{\frac{1}{2}}\right)$. For the complete determination of these expressions we add the constant factor of the ionic viscosity coefficient η_0 [see (2.4) and (2.12)]:

$$\eta_0=0.81M^3e^{-4}L^{-1}, \tag{5.4}$$

where L is the Coulomb logarithm (in accordance with [17] L = 2Λ, where Λ is the conventional Coulomb logarithm; see, e.g., [19]) and e is the elementary electrical charge.

For a cylindrically symmetric converging shock wave (ν = 1) with self-similarity index k = 1.226 and in the case of a deuteron plasma $(M/m)^{1/2}$ = 60.5 this problem has been solved numerically in [16]. The boundary conditions in this case are formulated according to (3.2), and the functions $X_1(\tau)$ and $\Pi(\tau)$ given by relations (3.4) and (3.5) are also borrowed from the self-similar solution. Certain details of the numerical solution may be found in [16]. The domain of appreciable deviation from the self-similar solution is characterized by $x\lesssim 0.1$. Here we write the values $\Theta_{i\,\max}=0.614$ and $\Theta_{e\,\max}=$ 0.268 ($u_{\max}\approx-0.95$, $\Pi_{\max}\approx 19$) of the maximum parameters of the plasma in the axial zone. In accordance with these data we write the maximum parameters at once in the system of units for the complete problem (4.1), generalizing relations (4.5) to the two-temperature case:

$$\Theta'_{i\max}=0.614\,(\xi'_0)^{-1.05}\alpha^{-0.237},\quad \Theta'_{e\max}=0.268\,(\xi'_0)^{-1.05}\alpha^{-0.237},$$

where the dimensionless parameter α is determined in (4.4) with s = 5/2 and η_0 from (5.4),

$$\alpha=1.08\frac{M^3}{e^4L}\frac{p_0'^2}{\rho_0^{3}r_0'}.$$

The constant of the self-similar solution in the system of units (4.1) $\xi'_0=-X_f'^{-1.226}\tau'_f$ must be specified from the results of numerical solution of the complete problem, in which dissipation effects are actually insignificant. If the quantity p'_0 taken as the pressure scale in (4.1) indeed characterizes the external impulse, i.e., is chosen in accordance with the boundary conditions of the complete problem, and r'_0 characterizes the radius of the system, then obviously this constant, as already mentioned, $\xi'_0\sim 1$, and quantitatively the degree of cumulation is mainly determined by the parameter α, which in general is very small ($\alpha\ll 1$). The maximum cumulation parameters for the velocity and pressure can be determined from Eqs. (4.6).

6. The statement of the problem of a converging shock wave in this article actually suggests three distinct stages in the cumulative process. First, a certain external impulse (externally applied pressure, moving piston, etc.) imparts motion to the gas in the given system (with set values of, say, the initial density and radius), during which time a self-similar converging shock wave is generated. The second stage entails its propagation toward the center or axis of the system according to a self-similar law. Dissipation effects do not play a significant role in these two stages. The third stage of the process, on the other hand, proceeds under the dominant influence of dissipation effects, the nature of which is determined by the properties of the gas or plasma. In this stage the front of the converging shock wave attains the center or axis of the system and is reflected from it. For known parameters of the self-similar solution we have formulated the gasdynamical problem with dissipation effects, corresponding to the third stage of motion. General relations have then been established between the results of solving that problem and the solution of the complete problem with the original initial and boundary conditions.

This subdivision of the complete problem could be avoided if it were possible to take correct account of the dissipation effects in all three of the indicated stages, for example if a universal numerical method of solution were available. But even in this unrealistic situation it would still be impossible to obtain general relations for the cumulation parameters, such that the use of numerical methods could be minimized. It also suffices to note that the delineation of the third stage (by analogy with the classical problem with a boundary layer in an ideal fluid), for which we have written out the system of equations (2.6)-(2.8) (gas) or (2.6), (5.1)-(5.3) (plasma) subject to the boundary conditions (3.2), demonstrates (and we speak cautiously here, at the physical level of rigor) that all cumulation parameters having a singularity in the self-similar solution are bounded. This assertion is most likely true in the most general case of a self-similar converging shock wave. In relation to the general cumulation problem discussed by Zababakhin [20], the case of self-similar converging shock waves, bearing in mind the indicated reservations, is therefore an example in which, contrary to [20], dissipation effects chosen by a physically justified procedure eliminate the unbounded growth of the cumulation parameters.

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